$\underline{Arithmetic\ cycles,\ Modular\ forms,\ and\ L\text{-}functions}}$ A celebration of the Mathematics pioneered by Henri Darmon

August 18-22, 2025, Room 1140, Pavillon André-Aisenstadt, Université de Montréal

Time	Mon.	Tues.	Wed.	Thurs.	Fri.
9:00 - 10:00	Registration	Gehrmann	Tian (Zoom)	Grossi	Tamiozzo (Zoom)
10:00 - 10:20	Opening remarks	Coffee break	Coffee break + Group photo	Coffee break	Coffee break
10:20 - 11:20	Bertolini	Beneish	Dembele	Barrera	Lilienfeldt
11:30 - 12:30	Imamoglu	Rotger (Zoom)	Rosu	Charollois (Zoom)	Prasanna
2:30 - 3:30	Skinner	Ciperiani		Liu	
3:30 - 4:00	Coffee break	Coffee break		Coffee break	
4:00 -5:00	Zerbes	Alfes		Sweeting	
	Reception	Joint Zoom activity with René 25 + Discussion panel on "Community building and mentoring"	Banquet (by invitation)		

Title and Abstracts

Monday, 18th August 2025

• Speaker: Massimo Bertolini (University of Duisburg-Essen).

Time: 10:20-11:20.

Title: *p*-adic Gross–Zagier formulas.

<u>Abstract:</u> I will report on old and new p-adic Gross–Zagier formulas, highlighting my collaboration with Henri Darmon.

• Speaker: Ozlem Imamoglu (ETH Zürich).

<u>Time:</u> 11:30-12:30.

Title: Special values of automorphic function on the hyperbolic 3 space.

<u>Abstract</u>: In this talk I will report on some recent results, joint with S. Herrero, M. Scwagenscheidt and A. von Pippich where we study the special values of automorphic functions for the hyperbolic 3 space and prove some algebraicity results for their averages over classes.

• Speaker: Christopher Skinner (Princeton University).

<u>Time:</u> 2:30-3:30.

Title: Euler systems, Iwasawa theory, and Special Value.

<u>Abstract</u>: I will report on some recent work on the arithmetic of special values of modular forms and related Euler systems. I will attempt to focus on how the threads of Darmon's ideas and results are woven through the tapestry.

• Speaker: Sarah Zerbes (ETH Zürich).

Time: 4:00-5:00.

Title: Euler systems via ultrafilters.

Abstract: In order to attack the Bloch—Kato conjecture and the Iwasawa Main Conjecture, one needs to control the size of Selmer groups (subspaces of Galois cohomology defined by local conditions). The basic machinery of Kolyvagin shows that any Euler system for a p-adic representation gives a bound on the "strict" Selmer group, i.e. the cohomology classes whose localization at p is 0. However, in practice one wants to bound a different, larger Selmer group with a less strict local condition; this is well understood for p-ordinary representations, but the general case is difficult, since it is very unclear how Kolyvagin's "derivative" operator interacts with local conditions.

Using a reinterpretation of Kolyvagin's construction via ultrafilters, we give a simple criterion for which local conditions interact well with the derivative construction. This has applications to the Bloch–Kato conjecture and to Pottharst's formulation of the Iwasawa Main Conjecture.

Tuesday, 19th August 2025

• Speaker: Lennart Gehrmann (Bielefeld University).

Time: 9:00-10:00.

<u>Title:</u> P-adic families of Funke–Millson theta series and applications.

Abstract: By considering a weighted version of Kudla and Millson's geometric theta series, Funke and Millson constructed modular forms with values in the cohomology of certain local systems on orthogonal symmetric spaces. We show that these modular forms can be p-adically interpolated. In particular, the Oda lift from holomorphic cusp forms on orthogonal Shimura varieties to elliptic modular forms can be put into p-adic families.

In the second half of the talk, we specialize to the case of definite ternary quadratic forms over totally real fields. We prove a formula relating higher-order derivatives of p-adic families of weighted theta series to p-adic logarithms of plectic points. This yields a Gross-Kohnen-Zagier theorem for plectic points.

This is joint work with Paul Kiefer respectively Michele Forna and Martí Roset.

• Speaker: Lea Beneish (University of North Texas).

Time: 10:20-11:20.

<u>Title:</u> How often does a cubic hypersurface have a rational point?

Abstract: A cubic hypersurface in \mathbb{P}^n defined over \mathbb{Q} is given by the vanishing locus of a cubic form f in n+1 variables. It is conjectured that whenever $n \geq 3$, a cubic hypersurface has only possibly local obstructions to solubility; this is now known to hold on average when $n \geq 4$ due to recent work of Browning, Le Boudec, and Sawin. Using this result, we determine the proportion of cubic hypersurfaces ordered by height with a rational point for $n \geq 4$ explicitly as a product over primes p of rational functions in p. In particular, we find that this proportion is equal to 1 for hypersurfaces in \mathbb{P}^9 . For 100% of cubic hypersurfaces, this recovers a celebrated result of Heath-Brown that non-singular cubic forms in at least 10 variables have rational zeros. This talk is based on joint work with Christopher Keyes.

• Speaker: Victor Rotger (Universitat Politècnica de Catalunya).

Time: 11:30-12:30.

Title: Darmon points and twisted triple-product *p*-adic *L*-functions.

<u>Abstract</u>: In this lecture I will explain work in progress with Henri Darmon and Alan Lauder concerning a twisted version of our elliptic Stark conjectures, relating the p-adic logarithm of a global point on an elliptic curve E/\mathbb{Q} to the first derivative of a triple-product p-adic L-function associated to E and a Hilbert modular eigenform of weight (1,1) over a real quadratic field in which p remains inert.

• Speaker: Mirela Ciperiani (University of Texas at Austin).

Time: 2:30-3:30.

Title: Shadow line distributions.

Abstract: Let E be an elliptic curve over \mathbb{Q} with Mordell-Weil rank 2 and p be an odd prime of good ordinary reduction. For every imaginary quadratic field K satisfying the Heegner hypothesis, there is a line in $E(K) \otimes \mathbb{Z}_p$ given by universal norms coming from the Mordell-Weil groups of subfields of the anticyclotomic \mathbb{Z}_p -extension of K, called the *shadow line*. When the twist of E by K has analytic rank 1, the shadow line is conjectured to lie in $E(\mathbb{Q}) \otimes \mathbb{Z}_p$; we verify this computationally in all our examples. We will discuss the distribution of shadow lines in $E(\mathbb{Q}) \otimes \mathbb{Z}_p$ as K varies and corresponding conjectures based on the computations we have made. This is joint work with Barry Mazur, Jennifer Balakrishnan, and Karl Rubin.

• Speaker: Claudia Alfes (Bielefeld University).

Time: 4:00-5:00.

<u>Title:</u> On harmonic weak Maass forms associated to even integer weight newforms.

<u>Abstract</u>: In this talk we review results on several types of harmonic weak Maass forms that are related to integral even weight newforms.

Starting with an integral weight newform, we will review different constructions of integral weight harmonic weak Maass forms via (generalized) Weierstrass zeta functions that map to the newform under the ξ -operator. A second construction via theta liftings gives a half-integral weight harmonic weak Maass form whose coefficients are given by periods of certain meromorphic modular forms with algebraic coefficients and periods of the integer even weight newform. This is joint work with Jens Funke, Michael Mertens, and Eugenia Rosu resp. Jan Bruinier and Markus Schwagenscheidt.

Wednesday, 20th August 2025

• Speaker: Lassina Dembele (King's College London).

<u>Time:</u> 9:00-10:00.

<u>Title:</u> Generalised Farey sets and fundamental domains of Bianchi groups.

Abstract: The Farey sequence and tessellation are important tools used to study the geometry of the hyperbolic 2-space. In this talk, we will introduce the notion of generalised Farey sets and use this to give a very beautiful description of a deformation retract of the hyperbolic 3-space (also known as the Mendoza retract). We will then explain how this can be used to compute presentations for Bianchi groups. Please come prepared! There will be lots of beautiful pictures! (Joint work with Sara Varljen.)

• Speaker: Ye Tian (Morningside Center of Mathematics).

Time: 10:20-11:20.

Title: Non-vanishing of L-values.

<u>Abstract:</u> We discuss non-vanishing of Rankin L-values over Iwasawa extensions. This is joint work with A. Burungale, W. He and X. Ye.

• Speaker: Eugenia Rosu (Mathematical Institute, Leiden University).

Time: 11:30-12:30.

<u>Title:</u> A higher degree Weierstrass function.

Abstract: The Weierstrass \wp -function plays a great role in the classical theory of complex elliptic curves. A related function, the Weierstrass ζ -function, is used by Guerzhoy to construct preimages under the ξ -operator of newforms of weight 2, corresponding to elliptic curves. In this talk, I will discuss a generalization of the Weierstrass ζ -function and an application to harmonic Maass forms. More precisely, I will describe a construction of a preimage of the ξ -operator of a newform of weight k for k > 2. This is based on joint work with C. Alfes-Neumann, J. Funke and M. Mertens..

Thursday, 21st August 2025

• Speaker: Giada Grossi (CNRS, Université Sorbonne Paris Nord).

Time: 9:00-10:00.

<u>Title:</u> From Asai to Triple Product: Euler Systems and *p*-adic *L*-functions.

<u>Abstract</u>: I will discuss recent work on Euler systems and *p*-adic *L*-functions for Hilbert modular forms. I will highlight how these developments are rooted in ideas introduced in previous works by Henri Darmon and his collaborators.

In the case of Asai motives attached to quadratic Hilbert modular forms, a Rankin–Selberg-type integral yields both the Asai–Flach Euler system and a p-adic L-function. I will outline how their relation, proved in joint work with D. Loeffler and S. Zerbes, leads to new cases of the Bloch–Kato conjecture.

I will also present ongoing work with A. Graham on the twisted triple product L-function. Ichino's integral and higher Hida theory play a central role in constructing a p-adic L-function in the "Hilbert-dominant region", with the goal of approaching higher-rank analogues of the Birch-Swinnerton-Dyer conjecture.

• Speaker: Daniel Barrera Salazar (Universidad de Santiago de Chile).

Time: 10:20-11:20.

Title: Local-global compatibility and the exceptional zero conjecture for GL(3).

Abstract: If E is a rational elliptic curve with split multiplicative reduction at p, then its p-adic L-function $L_p(E,s)$ has an "exceptional zero" at s=1 regardless of the vanishing of the complex L-value L(E,1). A seminal result of Greenberg and Stevens gives a precise formula for the first derivative of $L_p(E,s)$ at s=1 in terms of the complex L-value L(E,1) and the so-called L-invariant.

In this talk, we will explain a generalisation of this exceptional zero formula to regular algebraic, cuspidal automorphic representations of $\mathrm{GL}(3)$ which are Steinberg at p. The proof is divided into two different parts. The automorphic part , inspired in works of Darmon and Spiess, establishes a formula relating the relevant p-adic L-function to the automorphic L-invariants defined by Gehrmann, and the Galois part follows the strategy of Greenberg—Stevens and Gehrmann—Rosso, employing techniques from the p-adic deformation of Galois representations, to get the equality of the L-invariants. A key ingredient is a local-global compatibility result for p-adic families of ordinary automorphic representations, for which we follow the arguments of the "10-author paper." This a joint work with Andrew Graham and Chris Williams.

• Speaker: Pierre Charollois (Sorbonne Université).

Time: 11:30-12:30.

<u>Title:</u> "Sur une analogie réelle de la multiplication complexe" — A Continuing Tale from Mathias Lerch to Henri Darmon.

<u>Abstract</u>: Thanks to Hugo Chapdelaine and his Czech colleagues, we have uncovered a previously unknown 1904 manuscript by M. Lerch, entitled "Sur une analogie réelle de la multiplication complexe." In this talk, I will discuss how Lerch's main formula and his prescient method are deeply connected to various modern approaches to Hilbert's 12th problem, including recent works by Henri Darmon and his collaborators.

• Speaker: Zheng Liu (University of California, Santa Barbara).

<u>Time:</u> 2:30-3:30.

<u>Title:</u> Hida families of Yoshida lifts.

<u>Abstract</u>: We construct a Hida family of Yoshida lifts for two given Hida families f, g of modular forms. Its congruence with Hida families of stable forms on GSp(4) gives rise a lower bound on the

Selmer group of the Rankin-Selberg product of f, g in terms of a two-variable p-adic L-function. We study this congruence by using Rallis inner product formula, Bessel periods and Furusawa's pullback formula.

• Speaker: Naomi Sweeting (Princeton University).

Time: 4:00-5:00.

Title: On the Bloch–Kato Conjecture for some four-dimensional symplectic Galois representations.

Abstract: The Bloch-Kato Conjecture predicts a relation between Selmer ranks and orders of vanishing of L-functions for certain Galois representations. In this talk, I'll describe results towards this conjecture in ranks 0 and 1 for the self-dual Galois representations that come from Siegel modular forms on GSp(4) with parallel weight (3,3). The key step is a construction of auxiliary ramified Galois cohomology classes, which then give bounds on Selmer groups; the ramified classes come from level-raising congruences and the geometry of special cycles on Siegel threefolds.

Friday, 22nd August 2025

• Speaker: Matteo Tamiozzo (Université Sorbonne Paris Nord).

Time: 9:00-10:00.

<u>Title:</u> Modular geodesics and real bi-algebraic geometry.

Abstract: I will discuss the question, partly inspired by the work of Darmon–Vonk, whether real quadratic geodesics in the upper half-plane behave as "special cycles". I will explain why real quadratic geodesics, as well as geodesics with rational endpoints, are indeed "special" from the viewpoint of real bi-algebraic geometry. Furthermore, I will describe bi-algebraic cycles on elliptic curves parametrized by such geodesics. This talk is partly based on joint work with Arshay Sheth.

• Speaker: David Lilienfeldt (Leiden University).

Time: 10:20-11:20.

Title: Heights of generalized Heegner cycles

Abstract: In the 1980s, Gross and Zagier obtained a formula expressing the heights of CM points on modular curves in terms of derivatives of certain Rankin-Selberg L-functions, leading to applications towards the Birch and Swinnerton-Dyer conjecture for elliptic curves. In this talk, I will present a formula for the heights of certain algebraic cycles first introduced by Bertolini, Darmon, and Prasanna. This formula generalizes the Gross-Zagier formula to higher dimensions and has applications to the Beilinson-Bloch-Kato conjectures, notably in the case of Jacobians with CM. This is joint work with Ari Shnidman.

• Speaker: Kartik Prasanna (University of Michigan).

Time: 11:30-12:30

Title: Representations of GL_2 over $\mathbb{Z}/p^n\mathbb{Z}$, hypergeometric polynomials and binomial congruences

Abstract: Let $R = \mathbb{Z}/p^n\mathbb{Z}$. The representation theory of $\operatorname{GL}_2(R)$ over R-modules is well studied for n=1 but there is not much known for n>1. In this talk, I will describe a single result in the $R=\mathbb{Z}/p^n\mathbb{Z}$ case, that hints at a larger story. The proof of this result is elementary and eventually reduces to verifying some explicit (but difficult to prove) binomial congruences. It suggests some interesting connections between the archimedean representation theory of $\operatorname{Sp}(4)$, hypergeometric polynomials and representations of GL_2 over p-adic rings. I will also explain the motivation for considering this problem, which came from our attempt to solve a certain p-adic differential equation on a Siegel modular variety. This is joint work with Atsushi Ichino.